Math 2550 - Homework # 7

Subspaces of $\mathbb{R}^n$

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1. In $\mathbb{R}^2$ consider the vector $\vec{a} = \langle -1, -1 \rangle$.
   (a) Show that $\{\vec{a}\}$ is a linearly independent set. Conclude that $\beta = [\vec{a}]$ is a basis for the subspace $W = \text{span}(\vec{a})$.
   (b) List 4 vectors in W.
   (c) Draw a picture of W.
   (d) What is the dimension of W?
   (e) Show that $\vec{v} = \langle 4, 4 \rangle$ is in W. Draw a picture of $\vec{v}$ and W.
   (f) Show that $\vec{v} = \langle 1, \frac{1}{2} \rangle$ is not in W. Draw a picture of $\vec{v}$ and W.

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2. In $\mathbb{R}^3$ consider the vectors $\vec{i} = \langle 1, 0, 0 \rangle$ and $\vec{k} = \langle 0, 0, 1 \rangle$.
   (a) Show that $\vec{i}, \vec{k}$ are linearly independent vectors. Conclude that $\beta = [\vec{i}, \vec{k}]$ is a basis for $W = \text{span}(\vec{i}, \vec{k})$.
   (b) List 4 vectors in W.
   (c) Draw a picture of W.
   (d) What is the dimension of W?
   (e) Show that $\vec{v} = \langle 3, 0, 2 \rangle$ is in W.
   (f) Show that $\vec{v} = \langle 1, 3, 4 \rangle$ is not in W.

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3. In $\mathbb{R}^3$, let $\vec{a} = \langle 1, 1, 1 \rangle$, $\vec{b} = \langle 1, 0, 0 \rangle$
   (a) Show that $\vec{a}, \vec{b}$ are linearly independent vectors. Conclude that $\beta = [\vec{a}, \vec{b}]$ is a basis for $W = \text{span}(\vec{a}, \vec{b})$.
   (b) List 4 vectors in W.

(c) What is the dimension of W?

(d) Show that $\vec{v} = \langle \frac{1}{2}, -3, -3 \rangle$ is in W.

(e) Show that $\vec{v} = \langle 1, 2, 3 \rangle$ is not in W.

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4. For each W in the given $\mathbb{R}^n$, (i) show that W is a subspace of $\mathbb{R}^n$, (ii) find a basis for W, (iii) determine the dimension of W, and (iv) list four vectors in W.

(a) $W = \{\ \langle x, y \rangle \mid 2x - y = 0\}$ in $\mathbb{R}^2$

(b) $W = \{\ \langle x, y, z \rangle \mid x - y + 2z = 0$ and $y + z = 0\}$ in $\mathbb{R}^3$

(c) $W = \{\ \langle x, y, z \rangle \mid 2x - 4y - 3z = 0\}$ in $\mathbb{R}^3$

(d) $W = \{\ \langle x, y, z, w \rangle \mid x - z + u = 0$ and $y + z - u = 0\}$ in $\mathbb{R}^4$
